Simulation of the deflection of the ice cover

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Generalized discrete Fourier transform

Almost-periodic Bohr functions mean all non-discontinuous complex-valued periodic functions, as well as all trigonometric polynomials of the form

$$A(t) = \sum_{i=1}^{\infty} a_n e^{i \lambda_n t} \quad \sum_{n=1}^{\infty} |a_n| < \infty$$

The set of almost-periodic functions is denoted by $\mathcal{A}_W$. The coefficients may also depend on $y: a_n = a(\lambda, y)$.

There is an average value

$$a(\lambda) = M\{A(t)e^{-i\lambda t}\} = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} A(t)e^{-i\lambda t} \, dt$$

which is different from zero for no more than a countable set of values $\lambda: \lambda_1, \lambda_2, ...: a(\lambda_n) = a_n \neq 0$

$W_0: a(\lambda) \to A(t)$ generalized discrete Fourier transform

$W_0^{-1}: A(t) \to a(\lambda)$ inverse Fourier transform.

Properties

$$W_0^{-1} \frac{d^p A(t)}{dt^p} = (i \lambda)^p a(\lambda)$$

$$W_0^{-1} \frac{\partial^p A(t, y)}{\partial t^p} = (i \lambda)^p a(\lambda, y),$$

$$W_0^{-1} \frac{\partial^p A(t, y)}{\partial y^p} = \frac{d^p a(\lambda, y)}{dy^p}.$$
Mathematical formulation of the problem

Consider an ice slab in the form of a strip of finite width.

To find the area of a strip $-\infty < x < +\infty$, $0 \leq y \leq 1$ a function $w = w(x, y)$, which is continuous together with its partial derivatives up to fourth order and satisfies the equation

$$\Delta^2 w + K^2 w = Q,$$

where $K^2 = \frac{k}{D}$, $Q = \frac{q}{D}$, $q = q(x, y)$ - external distributed load, $k$ - stiffness coefficient of the elastic base, $D$ – coefficient of subgrade reaction, if the boundary conditions are given:

$$w\big|_{y=0} = 0, \quad w\big|_{y=1} = 0,$$

$$\frac{\partial w}{\partial y}\big|_{y=0} = 0, \quad \frac{\partial w}{\partial y}\big|_{y=1} = 0.$$
In case of cylindrical bending, the deflection function has the form

\[ w(y) = a_0(y) + d_1 e^{ny} \cos ny + d_2 e^{ny} \sin ny + d_3 e^{-ny} \cos ny + d_4 e^{-ny} \sin ny. \]

Figure shows a graph of the deflection function. The bending load has the form

\[ Q(y) = 4 + 16y^2, \quad K = 2. \]