MODEL OF LOCAL OPTIMAL CONTROL
FOR TECHNOLOGICAL MODES IN ELECTRIC POWER ASSOCIATIONS

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ELECTRIC POWER ASSOCIATION BASIC MODEL:

\[
\dot{\phi}_i = \omega_i,
\]

\[
\dot{\omega}_i = -\frac{1}{T_{i}^{(M)}} \sum_{j=1}^{n} \rho_{ij} (\varphi_i - \varphi_j) - \frac{R_i}{T_{i}^{(M)}} \omega_j + \frac{1}{T_{i}^{(M)}} p_i - \frac{1}{T_{i}^{(M)}} \mu_i,
\]

\[
\dot{p}_i = -\frac{1}{T_{i}^{(S)}} p_i + \frac{1}{T_{i}^{(S)}} q_i,
\]

\[
\dot{q}_i = -\frac{k_{ei}}{T_{i}^{(C1)}} \omega_i - \frac{1}{T_{i}^{(C1)}} q_i + \frac{1}{T_{i}^{(C1)}} \sigma_i,
\]

\[
\dot{\sigma}_i = -\frac{1}{T_{i}^{(C2)}} \sigma_i + \frac{1}{T_{i}^{(C2)}} u_i.
\]

The tie-line power flow between of the \( i \)-th and \( j \)-th EPA areas:

\[
S_{ij} = \sum_{j=1, j \neq i}^{n} \rho_{ij} (\varphi_i - \varphi_j).
\]

Variables:
- \( \varphi_i \) – deviation of the absolute angle of the \( i \)-th rotor from the base angle;
- \( p_i \) – the power transmitted to the \( i \)-th generator;
- \( \mu_i \) – unplanned power load.
- \( q_i \) - heat carrier (steam) power
- \( \sigma_i \) - secondary governor signal
- \( u_i \) - external control signal

Parameters:
- \( T_{i}^{(M)} \) – reduced mechanical inertia constant,
- \( R_i \) – rotor damping constant;
- \( \rho_{ij} \) – specific sync momentum (power) between \( i \)-th and \( j \)-th equivalent units (areas).
- \( T_{i}^{(S)} \) - steam-power time constant.
- \( T_{i}^{(C1)} \) - primary turbine governor time constant,
- \( k_{ei} \) - turbine speed primary governor gain.
- \( T_{i}^{(C2)} \) - secondary turbine governor time constant.
DISCRETE TIME LINEAR MODEL:

Turbine governors provide a quasi steady state mode

\[ \dot{p} = 0; \quad \dot{q} = 0; \quad \dot{\sigma} = 0 \]

Power and frequency depend on control signal:

\[ p_i = k_c \omega_i + u_i \]

Discrete time equation:

\[ X(k + 1) = HX(k) + F_u U(k) - F_m M, \quad Y(k) = CX(k), \quad X(0) = X_0. \]

\[ X = [\varphi_1, \ldots, \varphi_n, \omega_1, \ldots, \omega_n]^T, \quad U = [u_1, \ldots, u_n], \quad M = [\mu_1, \ldots, \mu_n]^T \]

\[ Y(k) : \text{frequencies and active power flows in the EPA tie-lines} - \text{linear combinations of} \ X(k) \ \text{components} \]

Current deviation of the state vector from the established steady state:

\[ x(k + 1) = Hx(k) + F_v u(k), \quad y(k) = Cx(k) \]

The task of the external control:

to calculate control signals \( u(k) \) to reduce vector \( y(k) \) to set values.
LFC CONTROL:

Area control error (ACE):

\[ \lambda_i(k) = (\omega_i(k) - \omega_0)(k_{ext} + R_i) + \sum_i (S_i(k) - S_{i0}) \]

\[ \lambda(k) = \sum \lambda_i(k) \]

ACE based control:

\[ \sum u_i(k) = K\lambda(k) , \]

Requirements to the control process:

validity:

restrictions on controls and tie-line flows deviations:

\[ u(k) \in [u_{\min}, u_{\max}] \]

\[ y(k+1) = Cx(k+1) \in [y_{\min}, y_{\max}] \]

optimality:

\[ J(k) = (y(k+1) - y_0)^T Q_1 (y(k+1) - y_0) + (u(k+1) - u_0)^T Q_2 (u(k+1) - u_0) \rightarrow \min \]

\( Q_1, Q_2 \) - weight matrices

\( y_0, u_0 \) - desired output and control values
LOCAL OPTIMAL CONTROL PROBLEM:

To calculate

\[ u(k) = \arg \min \{ J(k) : Iu(k) = K\lambda(k); Cx(k + 1) = y(k), u(k) \in [u_{\min}, u_{\max}] \} \]

Quadratic-linear programming problem:

Extended state vector:

\[ z(k) = \begin{pmatrix} y(k + 1) \\ u(k) \end{pmatrix} \]

Linear equation:

\[ Az(k) = \begin{pmatrix} E & -CF_U \\ 0 & I \end{pmatrix} \begin{pmatrix} y(k + 1) \\ u(k) \end{pmatrix} = \begin{pmatrix} CHx(k) \\ K\lambda(k) \end{pmatrix} = b(k) \]

Validity requirements:

\[ z(k) \in [z_{\min}, z_{\max}], \text{ where } z_{\min} = \begin{pmatrix} y_{\min} \\ u_{\min} \end{pmatrix}, \quad z_{\max} = \begin{pmatrix} y_{\max} \\ u_{\max} \end{pmatrix} \]

Optimality requirement:

\[ J(k) = (z(k) - z_0)^T Q(z(k) - z_0) \rightarrow \min \]

The task of the local optimal control:

\[ z(k) = \arg \min \{ J(k) : Az(k) = b(k), z(k) \in [z_{\min}, z_{\max}] \} \]
NORM MINIMIZATION ON THE INTERSECTION OF HYPERPLANE AND SPHERE:

\[ z = G\tilde{z} \]

\[ J(k) = ||\tilde{z}(k) - \tilde{z}_0||^2 \]

Parallelepiped approximated by inscribed sphere: \[ ||\tilde{z}(k) - \tilde{z}_c|| \leq R \]

Problem:

to find the point \( \tilde{z}(k) \) of Euclidean space closest to the point \( \tilde{z}_0 \) and laying within the intersection of the linear manifold (hyperplane) \( \tilde{A}\tilde{z}(k) = b(k) \) and sphere \( ||\tilde{z}(k) - \tilde{z}_c|| \leq R \), that is, to find the projection of the point \( \tilde{z}_0 \) on the hyperplane-sphere intersection.

Solution: projection operator (norm minimization operator), can be calculated analytically.

\[ \tilde{z}(k) = P(z_0, z_c, R, \tilde{A}, b(k)) \]

Nonlinear feedback operator:

\[ u(k) = \Phi(x(k)) \]

Lipshitz condition is fulfilled:

\[ ||\Phi(x) - \Phi(y)|| \leq L_\Phi ||x - y|| \]

\( L_\Phi \) depends on K factor
CONTROL PROCESS STABILITY:

discrete time equation with feedback:

\[ x(k + 1) = Hx(k) + B_U \Phi(x(k)) \]

fixed point:

\[ x^* = Hx^* + B_U \Phi(x^*) \]

Deviation from the fixed point:

\[ \Delta x(k + 1) = x(k + 1) - x^* = H(x(k) - x^*) + B_U (\Phi(x(k)) - \Phi(x^*)) = H\Delta x(k) + B_U (\Phi(x(k)) - \Phi(x^*)) \]

\[ \| \Delta x(k + 1) \| \leq \| H \| \cdot \| \Delta x(k) \| + \| B_U \| L_{\Phi} \| \Delta x(k) \| = (\| H \| + L_{\Phi} \| B_U \|) \| \Delta x(k) \| \]

Lyapunov asymptotic stability:

if \( \| H \| + L_{\Phi} \| B_U \| < 1 \) (which depends on K factor) then

\[ \Delta x(k) \to 0 \]