The comparison of several approaches to the interpolation of a trajectory of a navigation satellite

Authors: E. D. Karepova and V. S. Kornienko
Speaker: V. S. Kornienko

Study objective: to find the most suitable type of polynomial, as well as its order, the time interval for the calculation at which the trajectory approximates with a sufficient degree of accuracy (< 1 mm), and the number of such polynomials needed to fully cover the orbit of the satellite (12 h orbit). In figure 1 the used interpolation templates are presented.

Here the concept of significant interval was introduced. The significant interval will be called the calculated interval started with the node \( k \) by length \( N - 2k - 1 \ h \), where \( N \) is the number of nodes, \( k \) is user-defined indentation, \( h \) is the interpolation step. The confidence interval is considered here because it is known that for interpolation problems the greatest error occurs at the ends of the calculated interval, so it is obvious suppose not to use some part of it to predict the trajectory of the satellite. These templates can be easily generalized to polynomials of other orders.
Test on $f = \sin x$ function, $x \in \pi/3, 7\pi/3$

**Test 1.** The polynomial order was chosen as the smallest possible of the proposed ones (11, 14 or 17), but taking into account the condition that the approximation error on the test function should not exceed $10^{12}$ for sinus function. The testing showed that for covering of segment of $2\pi$ length the best variants are that use 4 polynomials of the 11-th order of 2, 3, 5.1 templates or 2 polynomials of the 14-th order of 1, 4 and 5.2 templates.

**Test 2.** Since the goal is the interpolation of satellite positions, additional restricts have to put in the interpolation problem. Firstly, the most accurate satellite positions are available with 15 minutes space. If we coarsely estimate the circuit time as 12 hours, then the step in the template should not be less than $\pi/24$. In numerical experiments, the step $\pi/6$ was chosen, which will correspond to a little less than an hour of satellite orbit. Secondly, the expected interpolation error on the test function should not exceed 10–12, which corresponds to the accuracy of share mm of an orbit.

In these figures the comparison of interpolation error between polynomial of 14-th order (except Hermite one, its order is 11) based on previously presented templates is provided on test function with the same interpolation step for all polynomials.
The error of the derivative approximation on sinus function after the numerical differentiation of these polynomials is practically the same and don’t exceed $10^{-11}$ that allows to approximate speeds of satellites with an accuracy of 1 mm/s.

Test 4. The recovery of the satellite position from the broadcast data. The velocities and acceleration was calculated by high-order Lagrange polynomial.

Test 3. The recovery of the satellite position from the data calculated using a mathematical model. We use the initial position and velocity of the satellite from a sp3 file. Further, according to the model of satellite motion, taking into account the non-sphericity of the geopotential and the influence of the Sun and the Moon, the ephemeris, velocity and speedup of the satellite were calculated. According to this data, the Hermite polynomial of the 11th degree was constructed with a template-size of 45 minutes. The interpolation error of the x-coordinate is shown on Figure. Here we use four glued Hermite polynomials (13 nodes) to approximation 9 hours of the satellite motion. The graph shows that the error does not exceed a share of mm. The differentiation of the interpolation polynomial allows to obtain the approximation of the satellite velocity at the same calculated interval with accuracy to 1 mm/s.